RFIC Design and Testing for Wireless Communications

A PragaTI (TI India Technical University) Course
July 18, 21, 22, 2008

Lecture 9: Frequency synthesizer design II (VCO)

By

Vishwani D. Agrawal
Fa Foster Dai

200 Broun Hall, Auburn University
Auburn, AL 36849-5201, USA

RFIC Design and Testing for Wireless Communications

Topics

Monday, July 21, 2008

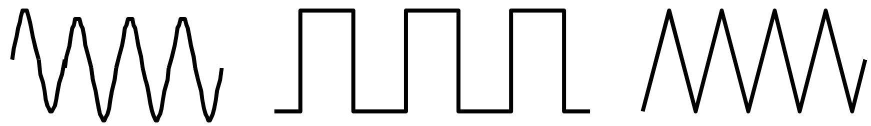
9:00 – 10:30	Introduction – Semiconductor history, RF characteristics
11:00 – 12:30	Basic Concepts – Linearity, noise figure, dynamic range
2:00 – 3:30	RF front-end design – LNA, mixer
4:00 – 5:30	Frequency synthesizer design I (PLL)

Tuesday, July 22, 2008

9:00 – 10:30	Frequency synthesizer design II (VCO)
11:00 – 12:30	RFIC design for wireless communications
2:00 – 3:30	Analog and mixed signal testing

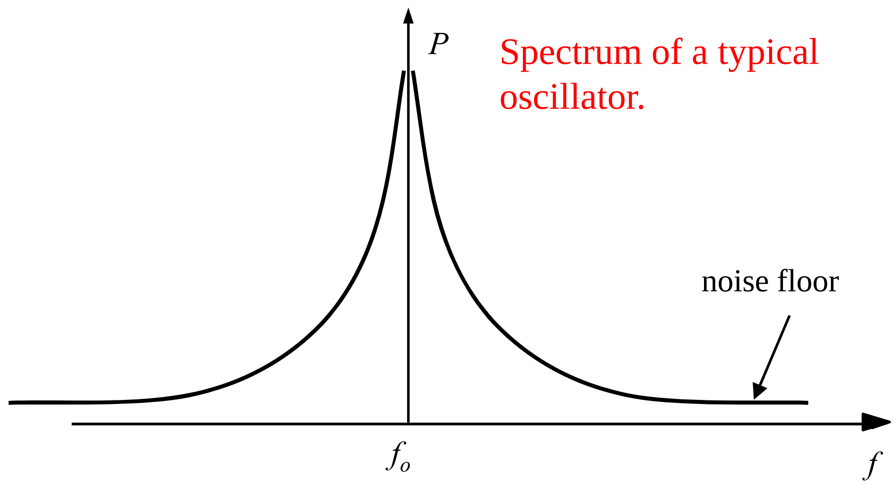
Phase Noise Specification of Oscillators

Example of oscillator periodic waveforms



- Phase noise. We desire accurate periodicity with all signal power concentrated in one discrete oscillator frequency → an impulse function in frequency domain. However, all real oscillators have less than perfect spectral purity and thus they develop "skirts". Power in the skirts is evidence of phase noise, Phase noise is any noise that charges the frequency or phase of the oscillator waveform. Phase noise is given by:

$$PN = \frac{p_o}{N_o}$$



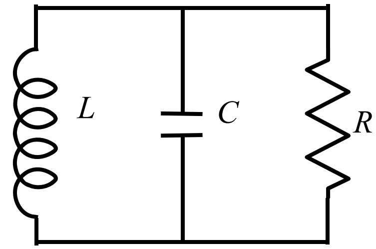
Spectrum of a typical oscillator.

- Where Po is the power in the tone at the frequency of oscillation and No is the noise power spectral density at some specified offset from the carrier. Phase noise is usually specified in *dBc/Hz*, meaning noise a 1-Hz bandwidth measured in decibels with respect to the carrier.

LC Resonator – Core of Oscillators

- If $i(t) = I_{pulse}\,\delta(t)$ is applied to a parallel resonator, the time domain response of the system can be found as:

$$v_{out} = \frac{\sqrt{2} I_{pulse} e^{\frac{-t}{2RC}}}{C} \cos\left(\sqrt{\frac{1}{LC} - \frac{1}{4R^2C^2}} \bullet t\right)$$

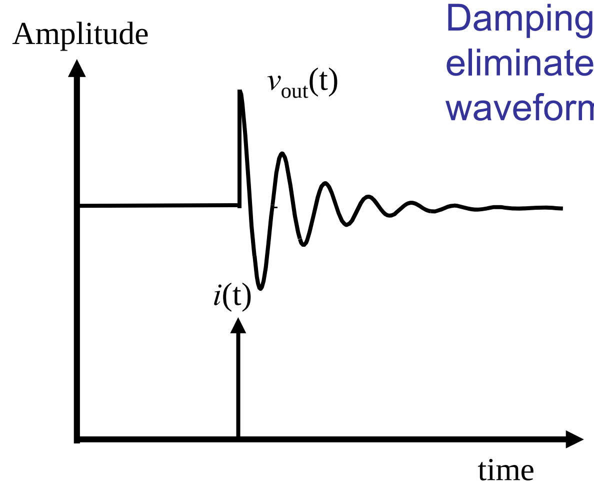


- Oscillation frequency

$$\omega_{OSC} = \sqrt{\frac{1}{LC} - \frac{1}{4R^2C^2}}$$

- In most oscillators, $|R| >> \sqrt{L/C}$

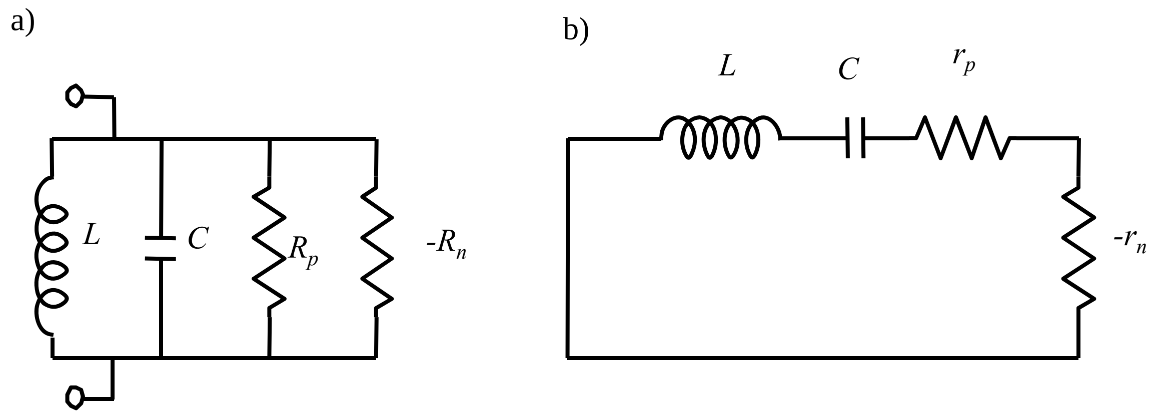
$$\omega_{OSC} = \sqrt{\frac{1}{LC}}$$



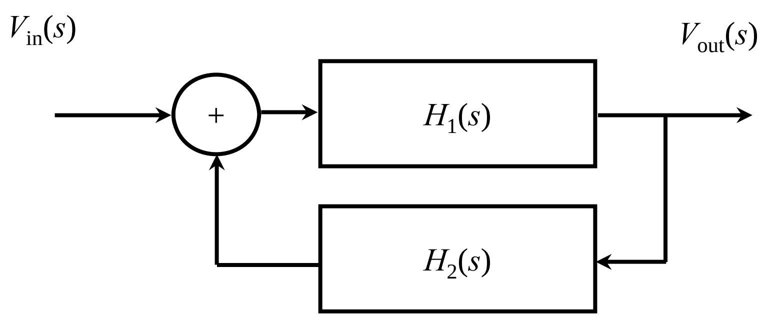
Damping must be eliminated for the waveform to persist!

Damped LC resonator with current step applied.

Adding Negative Resistance Trough Feedback to Resonator



The addition of negative resistance to the circuit to overcome losses in a) a parallel resonator or b) a series resonator.



Linear model of an oscillator as a feedback control system.

Barkhausen Criterion

- Closed loop gain

$$\frac{V_{out}(s)}{V_{in}(s)} = \frac{H_1(s)}{1 - H_1(s)H_2(s)}$$

- Condition for oscillation: denominator approaches zero. → To find the closed-loop poles

$$1 - H_1(s)H_2(s) = 0$$

- For sustained oscillation at constant amplitude, the pole must be on the *jω* axis. For the open-loop analysis

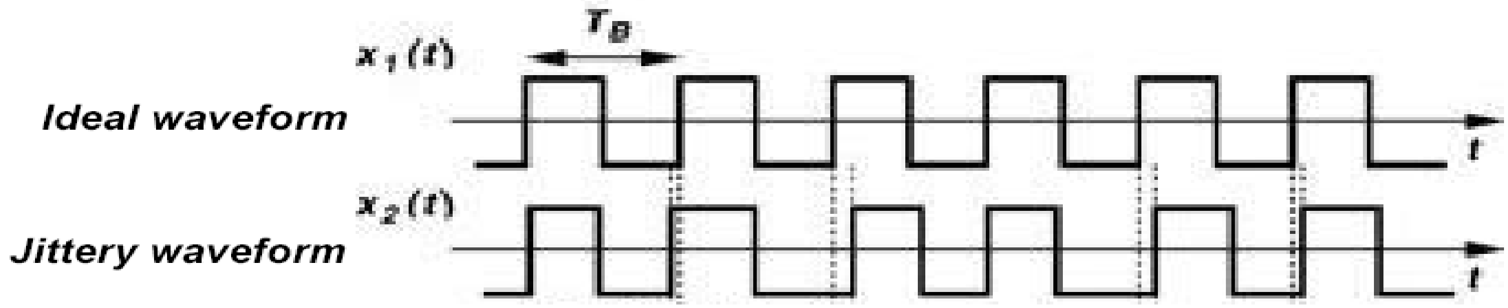
$$H_1(j\omega)H_2(j\omega) = 1$$

positive feedback with gain larger than or equal to 1.

- **Barkhausen criterion, which states that for sustained oscillation at constant amplitude, the gain around the loop is 1 and the phase around the loop is 0 or some multiple of 2π.**

$$|H_1(j\omega)||H_2(j\omega)| = 1 \qquad \angle H_1(j\omega)H_2(j\omega) = 2n\pi$$

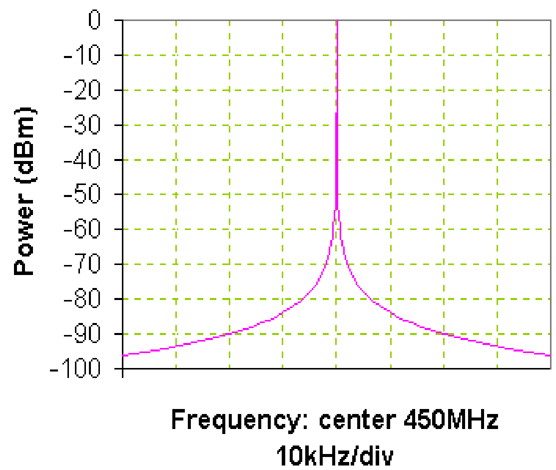
VCO Output Spectral Purity



Noise in one sideband in a 1Hz bandwidth:

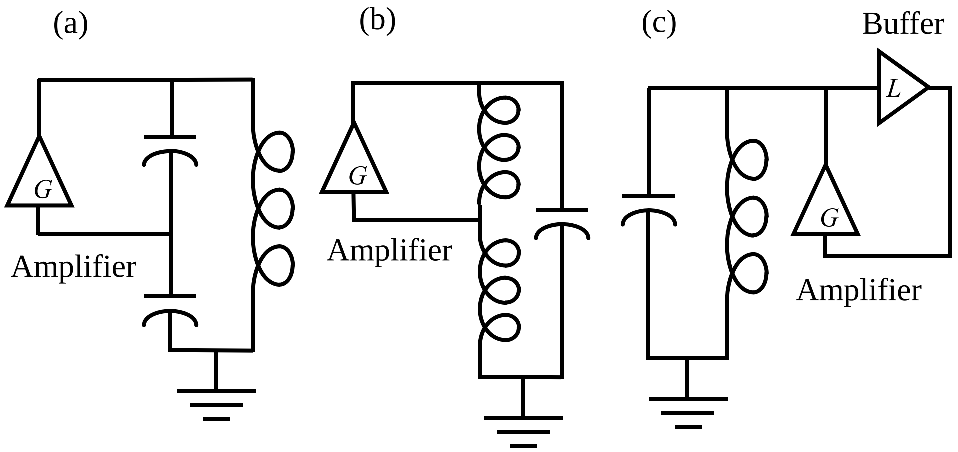
$$L_{total}(\Delta w) = \frac{\text{noise power in 1Hz BW at } w_o + \Delta w}{\text{Carrier power}}$$

Units: dBc/Hz

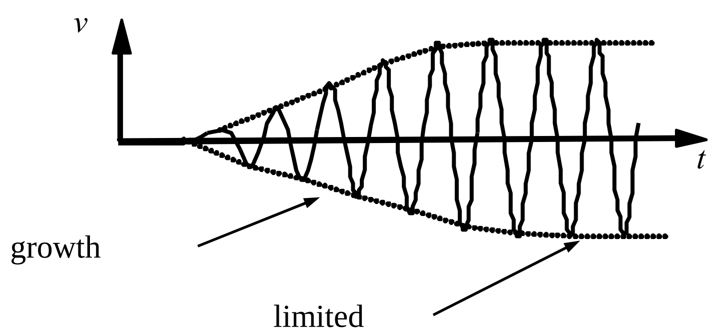


VCO Output Spectrum

Popular Implementation of Feedback to Resonator



Resonators with feedback. (a) Colpitts Oscillator.
(b) Hartley Oscillator (not suitable for IC). (c) $-G_m$ oscillator.

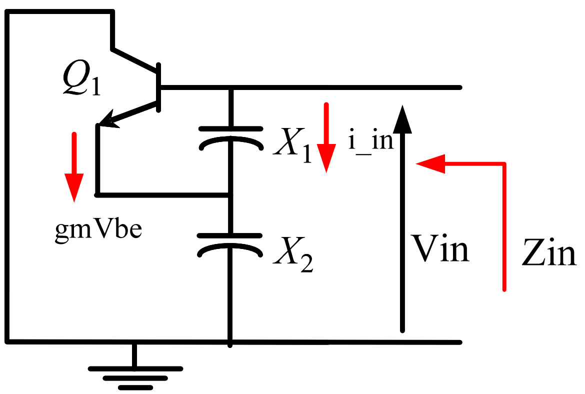


Waveform of an LC resonator with losses compensated. The oscillation grows until a practical constraint limits the amplitude.

Negative Resistance of Colpitts Amplifier

$$v_{in} = i_{in}\left(jX_1 + jX_2\right) + g_m v_{be}\left(jX_2\right)$$

Negative Impedance, if X_1 and X_2 are the same type



$$Z_{in} = \frac{v_{in}}{i_{in}} = -g_m X_1 X_2 + j\left(X_1 + X_2\right)$$

$$Z_{in} = -\frac{g_m}{\omega^2 C_1 C_2} + \frac{1}{j\omega C_1} + \frac{1}{j\omega C_2}$$

Load reactance needs to equal $-j(X_1+X_2)$ → add an inductor

To start the oscillation,

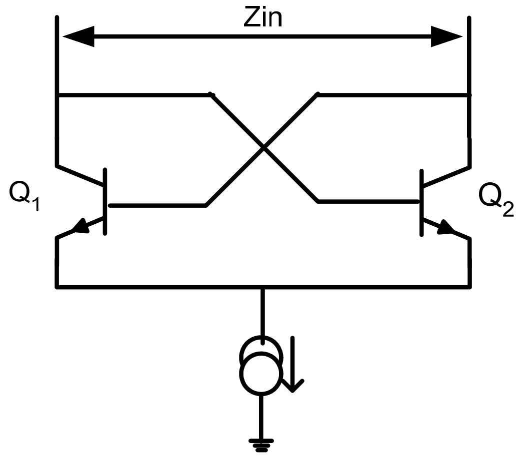
$$-g_m X_1 X_2 > R_L$$

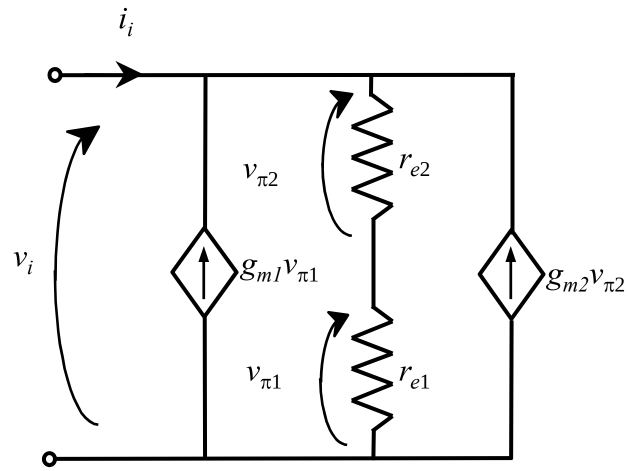
$$g_m > R_L \omega^2 C_1 C_2$$

Oscillation frequency

$$f_{osc} = \frac{1}{2\pi}\left[\frac{1}{L_3}\left(\frac{1}{C_1} + \frac{1}{C_2}\right)\right]^{1/2}$$

Negative Resistance of –Gm Oscillator





$$i_i = \frac{v_i}{r_{e1} + r_{e2}} = -g_{m1}v_{\pi 1} - g_{m2}v_{\pi 2}$$

- Both transistors are biased identically, $Z_i = \dfrac{-2}{g_m}$

- Condition for oscillation is that $g_m > \dfrac{2}{R_p}$

where Rp is the equivalent parallel resistance of the resonator.

VCO Mathematical Model

Output frequency of an ideal VCO:

$$w_{out} = w_{FR} + K_{vco} V_c$$

Sinusoidal output: $y(t) = A\cos(w_{FR} t + K_{vco} \int_{-\infty}^{t} V_C)$

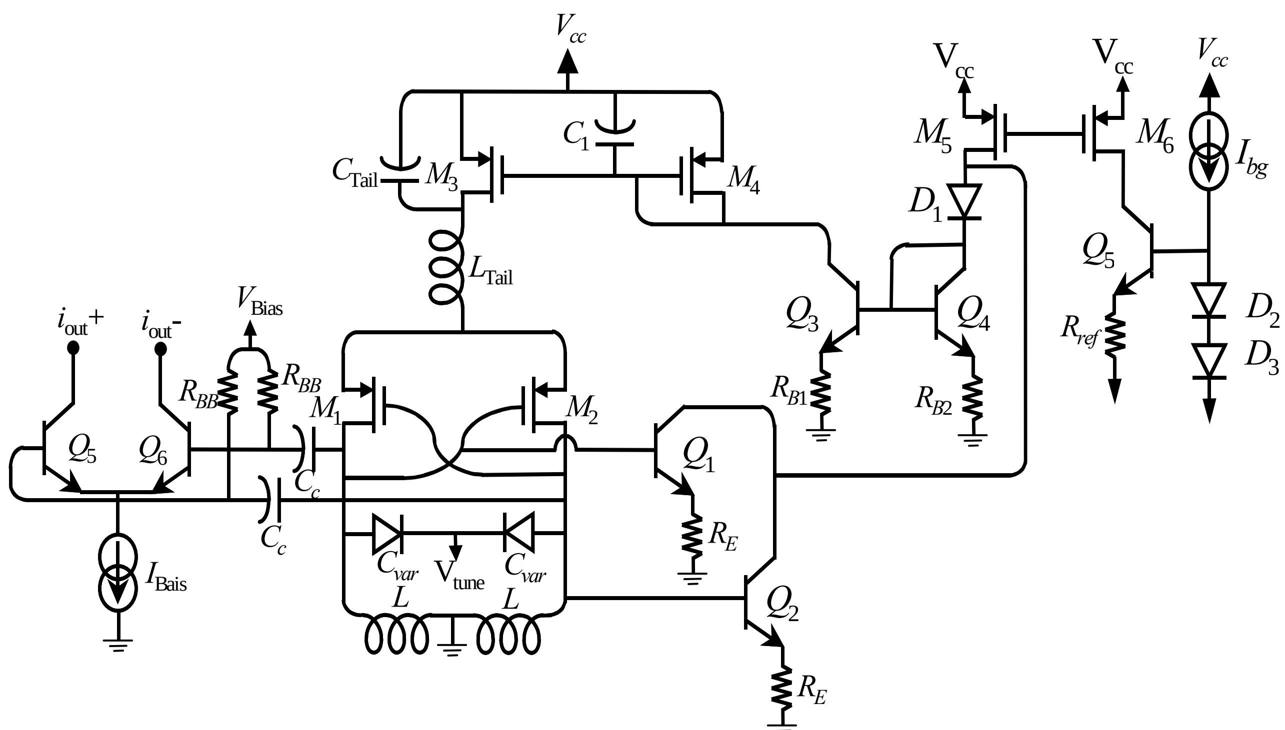
Open loop Q : $Q = \frac{w_0}{2} \left| \frac{d\phi}{dw} \right|$

w_0 is the center frequency

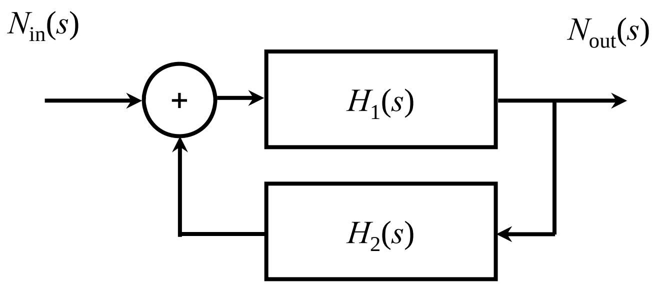
ϕ is the phase of open loop transfer function

PMOS VCO with Automatic Amplitude Control

- Large V_{tune} range (almost from 0V ~ Vcc V).
- Tank can be connected to ground rather than DC → lower phase noise and diodes can be connected in the proper polarity without additional biasing.
- PMOS transistors can be operated into saturation without affecting the VCO noise performance → higher output swing than bipolar VCO.
- High phase noise below 100kHz offset (due to high flicker noise) → can be tolerated by wider loop bandwidth (>100kHz).



Linear or Addictive Phase Noise -- Leeson's Formula



Oscillator Phase Noise $\Phi_n(t)$

$$V_{OSC} = A\cos[\omega_0 t + \phi_n(t)]$$

- Noise close loop transfer function $\frac{N_{out}(s)}{N_{in}(s)} = \frac{H_1(s)}{1 - H(s)}$
- Open loop transfer function $H(s)=H_1(s)H_2(s)$ $$H(j\omega) \approx H(j\omega_0) + \Delta\omega \frac{dH}{d\omega}$$
- Oscillation conditions $H(j\omega_0) = 1 \rightarrow H_1(j\omega_0) = H_1$
- Noise power $$\left|\frac{N_{out}(s)}{N_{in}(s)}\right|^2 = \frac{|H_1|^2}{(\Delta\omega)^2 \left|\frac{dH}{d\omega}\right|^2}$$

Oscillator Phase Noise – Leeson's Equation

$$H(\omega) = |H| e^{j\phi}$$

$$\frac{dH}{d\omega} = \frac{d|H|}{d\omega} e^{j\phi} + |H| j e^{j\phi} \frac{d\phi}{d\omega}$$

Orthogonal

ignorable dominant

$$\left|\frac{dH}{d\omega}\right|^2 = \left|\frac{d|H|}{d\omega}\right|^2 + |H|^2 \left|\frac{d\phi}{d\omega}\right|^2$$

- **At resonance, the phase changes much faster than magnitude, and |H|=1 near resonance. → ignore amplitude noise and AM to PM conversion as well.**

$$\left|\frac{dH}{d\omega}\right|^2 = \left|\frac{d\phi}{d\omega}\right|^2 \longrightarrow \left|\frac{N_{out}(s)}{N_{in}(s)}\right|^2 = \frac{|H_1|^2}{(\Delta\omega)^2 \left|\frac{d\phi}{d\omega}\right|^2}$$

$$Q = \frac{\omega_0}{2}\left|\frac{d\phi}{d\omega}\right| \longrightarrow \left|\frac{N_{out}(s)}{N_{in}(s)}\right|^2 = \frac{|H_1|^2 \omega_0^2}{4Q^2 (\Delta\omega)^2}$$

Oscillator Phase Noise – Leeson's Equation

- **If feedback path is unity, then H_1=H, and since |H|=1 near resonance**

$$\left|\frac{N_{out}(s)}{N_{in}(s)}\right|^2 = \frac{\omega_0^2}{4Q^2(\Delta\omega)^2}$$

- **Phase noise is quoted as an absolute noise referred to the carrier power**

$$PN = \frac{|N_{out}(s)|^2}{2P_S} = \left(\frac{|H_1|\omega_0}{(2Q\Delta\omega)}\right)^2\left(\frac{|N_{in}(s)|}{2P_S}\right)$$

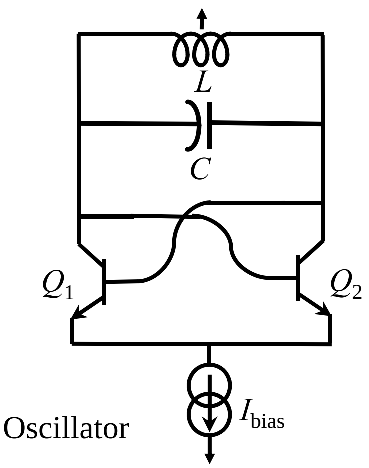
- **Ps is the signal power at active device input.**
- **If the transistor and bias were noiseless, then the only noise present would be due to the resonator losses. The transistors and the bias will add noise to the minimum noise of**

$$|N_{in}(s)|^2 = kT$$

Oscillator Phase Noise – Leeson's Equation

Active Device noise:

- **If ρ is the fraction of cycle for which the transistors are completely switched, in_t is the noise current injected into the oscillator from the bias during this time.**
- **During transitions (1- ρ), the transistors act like an amplifier, and collector shot noise i_{cn} usually dominates.**



c) -G_m Oscillator

$$|N_{in}(s)|^2 = kT + \frac{i_{nt}^2 R_p}{2}\rho + i_{cn}^2 R_p (1-\rho)$$

Resonator loss (source)

Bias noise

Transistor shot noise

$$F = 1 + \frac{i_{nt}^2 R_p}{2kT}\rho + \frac{i_{cn}^2 R_p (1-\rho)}{kT}$$

Rp is the equivalent parallel resistance of the tank.

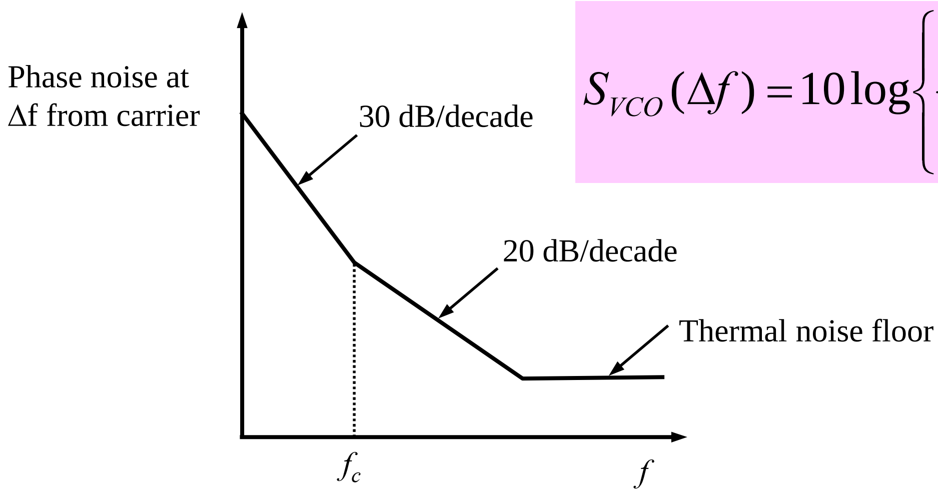
$$PN = \left(\frac{|H_1|\omega_0}{(2Q\Delta\omega)}\right)^2 \left(\frac{FkT}{2Ps}\right)$$

Oscillator Phase Noise – Leeson's Equation

- **It has been assumed that flicker noise is insignificant at the frequencies of interest. This may not be the case for CMOS designs. If ωc represents the flicker noise corner where flicker noise and thermal noise are equal, phase noise is given by**

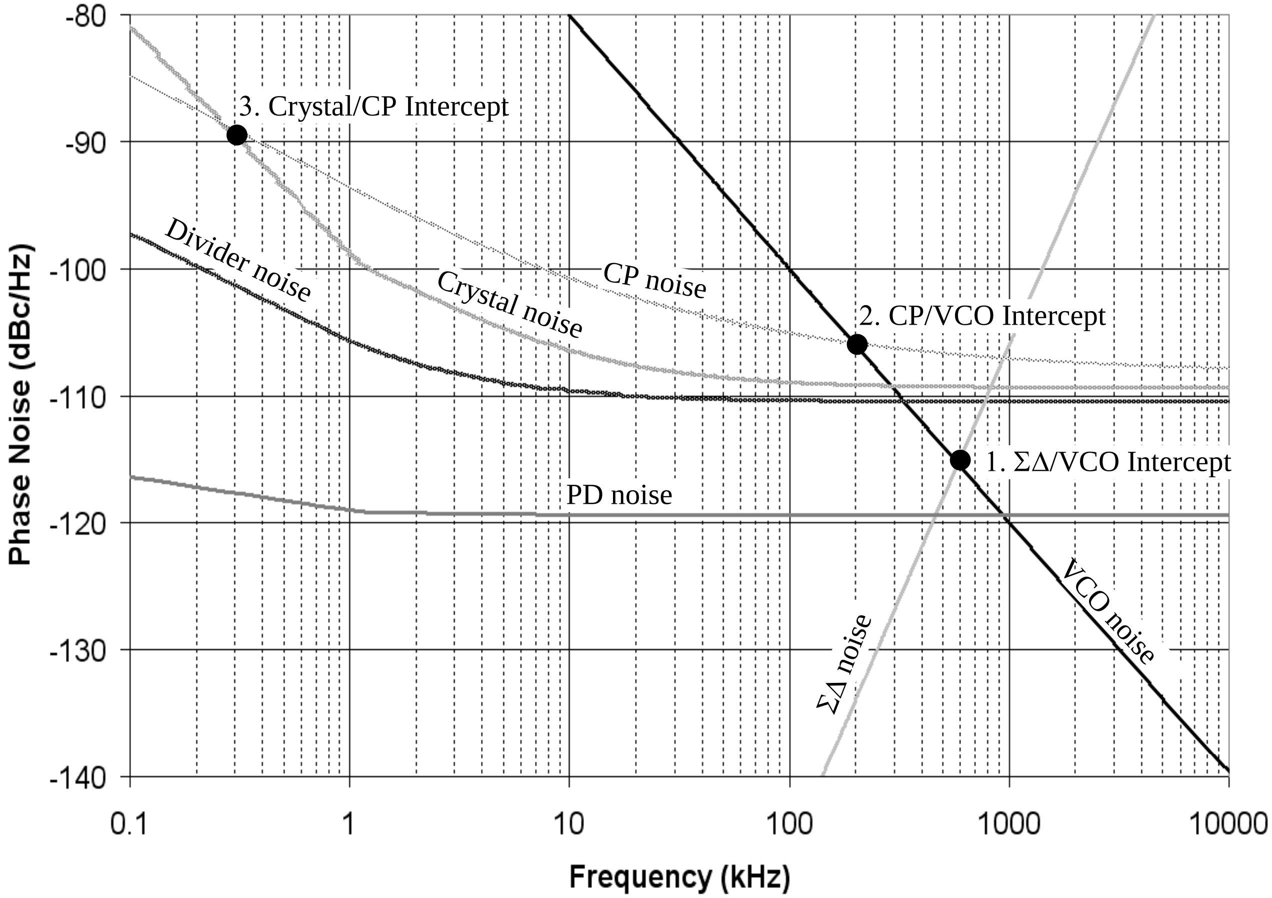
$$PN = \left(\frac{|H_1|\omega_0}{(2Q\Delta\omega)}\right)^2 \left(\frac{FkT}{2Ps}\right)\left(1+\frac{\omega_c}{\Delta\omega}\right)$$

- **Assuming unity feedback, oscillator output spectrum density**

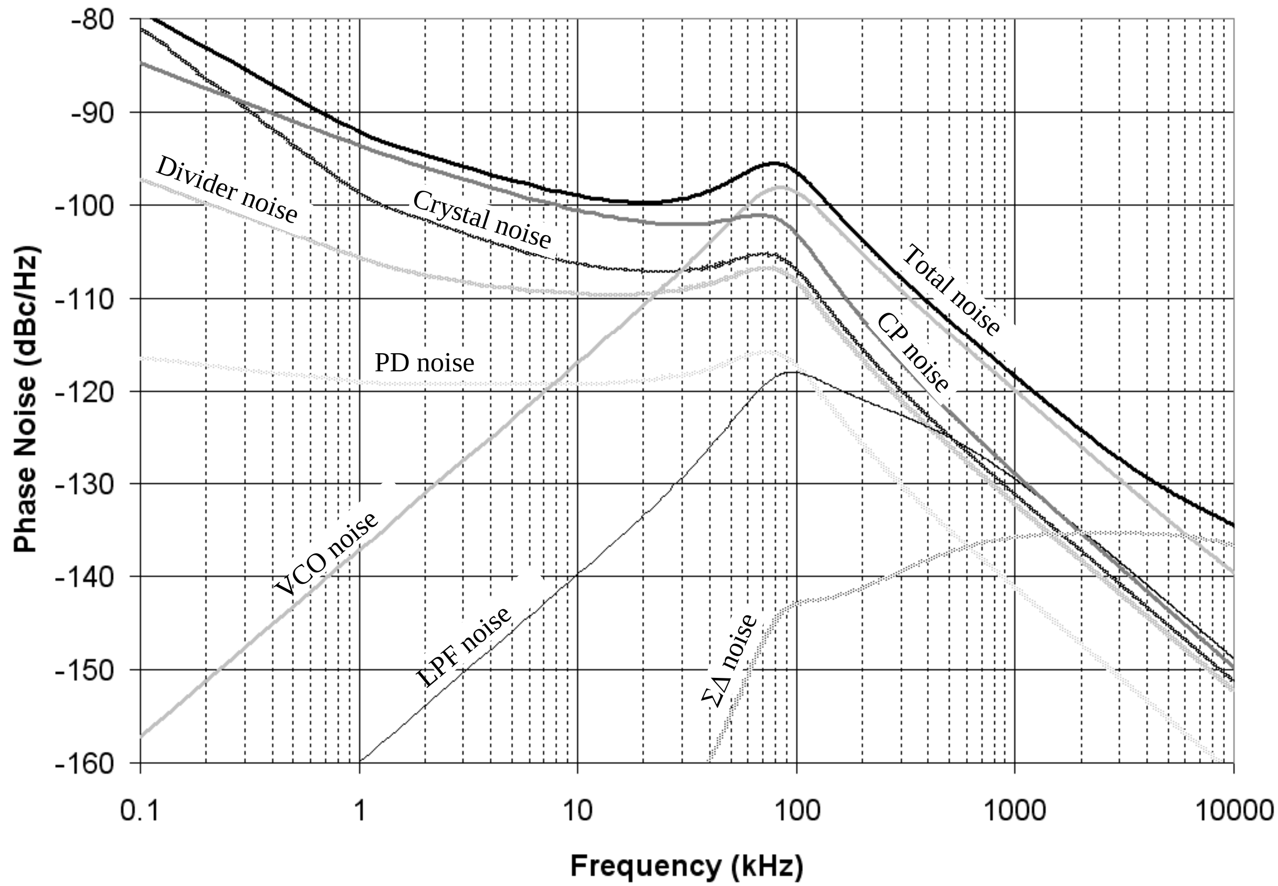


$$S_{VCO}(\Delta f) = 10\log\left\{\frac{FkT}{2P_s}\left[1+\left(\frac{f_0}{2\Delta f \cdot Q_L}\right)^2\right]\cdot\left[1+\frac{f_c}{|\Delta f|}\right]\right\}$$

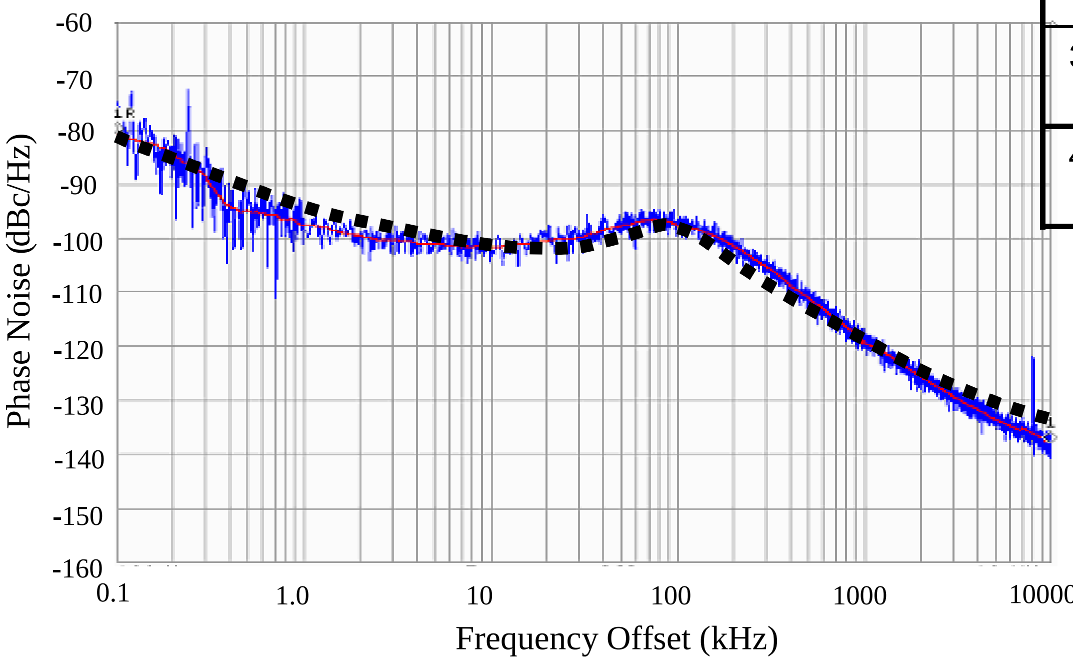
Simulated PLL Phase Noise Sources



Simulated PLL Phase Noise With Loop Effect



Comparison of measured and simulated phase noise



Frequency Band	Simulated Phase Noise	Measured Phase Noise
3.2-3.3GHz	0.44°rms	0.50°rms
4.1-4.3GHz	0.50°rms	0.535°rms

Parameter	Value
C_1	3nF
C_2	600pF
R	600Ω